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Quantum Gravitational Contributions to the Standard Model Effective Potential and Vacuum Stability

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Abstract

We compute the quantum gravitational contributions to the standard model effective potential and analyze their effects on the Higgs vacuum stability in the framework of effective field theory. Einstein gravity necessarily implies the existence of higher dimension ϕ^6 and ϕ^8 operators with novel couplings $\eta_{1/2}$ in the Higgs sector. The beta functions of these couplings are established and the impact of the gravity induced contributions on electroweak vacuum stability is studied. We find that the true minimum of the standard model effective potential now lies below the Planck scale for almost the entire parameter space ($\eta_{1/2}(m_\mathrm{t}) > 0.01$). In addition quantum gravity is shown to contribute to the minimal value of the standard model NLO effective potential at the percent level. The quantum gravity induced contributions yield a metastable vacuum for a large fraction of the parameter space in the flowing couplings $\eta_{1/2}$.

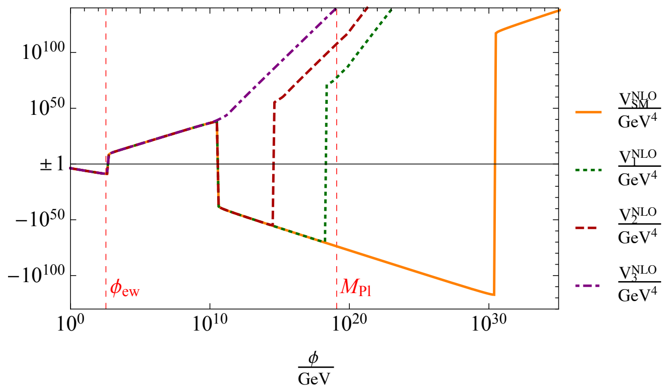

Figure 1: Double logarithmic plot of the NLO effective potential with and without gravitational corrections for sample initial values: (1) green/dotted: $\eta_1(m_{\text{t}}) = 0.5$, $\eta_2(m_{\text{t}}) = 0.3$ with minimal value $V_{1,\text{min}} = -4.9 \times 10^{69}$ GeV^4; (2) red/dashed: $\eta_1(m_{\text{t}}) = 10^7$, $\eta_2(m_{\text{t}}) = 10^9$ with $V_{2,\text{min}} = -4.6 \times 10^{54}$ GeV^4; (3) purple/dashdotted: $\eta_1(m_{\text{t}}) = 10^{15}$, $\eta_2(m_{\text{t}}) = 10^{20}$ where the minimum has disappeared. (SM) orange: Without gravity, we find $V_{\text{SM,min}} = -4.0 \times 10^{110}$ GeV^4.

A central outcome of the recent Higgs boson discovery [1, 2] and the absence of new physics signals at the LHC is that the standard model (SM) as a quantum field theory stays perturbatively self-consistent all the way up to the Planck scale M_{Pl} [3, 4]. From this perspective the conservative scenario of having no beyond-SM-physics up to M_{Pl}—except for gravity—is viable and has attracted attention [5–10].[1] Moreover, the measured values for the Higgs pole mass m_{H} and the top mass m_{t} have an intriguing consequence for the question of stability of the Higgs vacuum: The SM lies close to the border of absolute electroweak vacuum stability and metastability. Vacuum stability is usually studied by determining the renormalization group improved effective Higgs potential $V(\phi)$: If $V(\phi)$ develops a negative minimum below the value V_{ew} of the electroweak minimum, the SM becomes unstable; if the inverse decay rate for tunneling from the false electroweak minimum at $\phi = \phi_{\text{ew}}$ to the second (true) minimum at ϕ_{min} is larger than the lifetime of our universe, then the SM is said to be metastable. The value at the minimum $V_{\text{min}} = V(\phi_{\text{min}}) < V_{\text{ew}}$ is very sensitive to the value of m_{H} and m_{t}. State of the art high precision perturbative calculations [3, 4] using the latest experimental data indicate that the SM has a negative metastable minimum. However, it lies deep within the Planck regime: The gauge invariant value of V_{min} at next-to-leading order (NLO) precision is $(-V_{\text{min}}^{\text{NLO}})^{1/4} \sim 10^{10}\, M_{\text{Pl}}$ [11]. While it is fascinating that the SM may be extrapolated to such high energy scales, it is obvious that quantum gravitational effects cannot be ignored any longer in these regimes—even for the conservative no-new-physics scenario. In consequence, the celebrated statement of metastability of the SM based on the above value for V_{min} is spurious.

This is the motivation for the present analysis. We adopt the conservative viewpoint of having no new physics up to M_{Pl} and study the quantum gravity contributions to the SM effective poten-

[1] Of course neutrino masses, dark matter and baryogenesis still require some (mild) extensions of the SM.

tial. This is done by treating Einstein gravity as an effective quantum field theory [12]. For scales below $M_{\rm Pl} = 1.22 \times 10^{19}$ GeV, the SM coupled to quantum gravity is perturbatively well defined albeit of limited predictability due to the necessity of including higher dimensional operators as counterterms at every loop order. In fact, the main impact of the gravitational contributions to the Higgs effective potential is that non-renormalizability induces higher dimension counter terms of the form $\frac{\eta_1}{M_{\rm Pl}^2}\phi^6$ and $\frac{\eta_2}{M_{\rm Pl}^4}\phi^8$ into the effective field theory, with a priory undetermined couplings η_1 and η_2. The addition of such higher dimension operators to the tree-level potential has been studied in [13, 14, 11] as a means to parametrize potential new physics effects on top of the SM. Here we show that these terms are necessarily present due to the existence of gravity. Even if one chooses the new couplings to be absent at low scales, they are turned on at high scales by the quantum gravity contributions to the renormalization group equations (RGE). The higher dimensional counter terms have a profound effect: For generic positive values of $\eta_{1/2}(m_{\rm t})$ the true minimum of the SM effective potential is pushed to sub-Planckian scales. It is generically metastable or even stable for a large range of initial values of $\eta_{1/2}$ at the scale $m_{\rm t}$. See Figure 1 for generic configurations of $V(\phi)$ with and without gravitational contributions. On top, quantum gravity effects contribute to the effective potential at NLO with orders of magnitude at the percent level.

Standard model coupled to Einstein gravity. We consider the standard model coupled to gravity with a cosmological constant Λ:[2]

$$\mathcal{L}^{\rm SM}_{\rm grav} = \frac{2}{\kappa^2}\sqrt{-g}\,R + \mathcal{L}_{\rm GB} + \mathcal{L}_{\rm F} + \sqrt{-g}\left(\Lambda + g^{\mu\nu}\,\partial_\mu H\,\partial_\nu H^\dagger + m^2\,|H|^2 - \lambda\,|H|^4\right). \tag{1}$$

Here κ denotes the dimensionful gravitational coupling constant related to Newton's constant $\kappa^2 = 32\pi G = M_{\rm Pl}^{-2}$. As we are interested in the quantum gravitational contributions to the Higgs effective potential at the one-loop order, it is sufficient to study the Higgs-gravity sector of the above model since the gauge bosons in $\mathcal{L}_{\rm GB}$ and the matter fermions in $\mathcal{L}_{\rm F}$ decouple at this leading perturbative order. Moreover, due to the non-renormalizability of gravity we will be forced to include higher dimension scalar operators as counterterms

$$\mathcal{L}_{CT} = \sqrt{-g}\left(-\eta_1\,\kappa^2\,|H|^6 - \eta_2\,\kappa^4\,|H|^8\right), \tag{2}$$

carrying their own dimensionless bare couplings η_1 and η_2. We seek the one-loop corrections to the SM effective potential $V(\phi)$, where we expand the Higgs doublet about the constant real background field ϕ with $H = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_1 + i\psi_1\\ \phi + \varphi_2 + i\psi_2\end{pmatrix}$ and $\{\varphi_i, \psi_i\} \in \mathbb{R}$. The tree-level Higgs potential is then given by

$$V^{\rm tree}(\phi) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{\eta_1}{8}\kappa^2\phi^6 + \frac{\eta_2}{16}\kappa^4\phi^8\,. \tag{3}$$

[2]Note that we could also include an additional non-minimally curvature-coupled scalar term $2\sqrt{-g}\,R\,\rho\,|H|^2$ into the Lagrangian. However, the freedom of Higgs-field redefinitions and Weyl rescalings may be used to set $\rho = 0$ in (1) [15] at the cost of introducing an additional kinetic term. Here we set $\rho = 0$ from the start.

In the gravitational sector we work in de Donder gauge expanding the metric field as $g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$. This yields the following standard expansions of the Einstein–Hilbert and de Donder gauge fixing terms $\mathcal{L}_{\text{g.f.}}$:

$$
\begin{aligned}
\frac{2}{\kappa^2}\sqrt{-g}R + \mathcal{L}_{\text{g.f.}} &= \tfrac{1}{2} h_{\alpha\beta}\, P^{\alpha\beta;\gamma\delta}\, \partial^2 h_{\gamma\delta} + \mathcal{O}(h^3), \\
\sqrt{-g} &= 1 + \frac{\kappa}{2}\, h_{\alpha\beta}\eta^{\alpha\beta} + \frac{\kappa^2}{4}\, h_{\alpha\beta}\, P^{\alpha\beta;\gamma\delta}\, h_{\gamma\delta} + \mathcal{O}(h^3), \\
P^{\alpha\beta;\gamma\delta} &= \tfrac{1}{2}(\eta^{\alpha\beta}\,\eta^{\gamma\delta} - \eta^{\alpha\gamma}\,\eta^{\delta\beta} - \eta^{\alpha\delta}\,\eta^{\gamma\beta})\,.
\end{aligned} \tag{4}
$$

For the quadratic fluctuations of the graviton and Higgs field in our model described by (1) we then have

$$
\begin{aligned}
\mathcal{L}^{\text{quad}}_{h_{\mu\nu},\text{H}} = &- \tfrac{1}{2} h_{\alpha\beta}\left[P^{\alpha\beta;\gamma\delta}\Big(- \partial^2 + m_A^2\Big)\right] h_{\gamma\delta} - h_{\alpha\beta}\left[\eta^{\alpha\beta}\, m_B^2\right]\varphi_2 \\
&- \tfrac{1}{2}\varphi_2\left[\partial^2 + m_C^2\right]\varphi_2 - \sum_{\Phi_I=\{\varphi_1,\psi_1,\psi_2\}} \tfrac{1}{2}\Phi_I\left[\partial^2 + m_D^2\right]\Phi_I\,,
\end{aligned} \tag{5}
$$

with the effective masses

$$
\begin{aligned}
m_A^2 &= \frac{\kappa^2}{4}\Big(- m^2\phi^2 + \tfrac{1}{2}\lambda\phi^4 + \tfrac{1}{4}\kappa^2\eta_1\phi^6 + \tfrac{1}{8}\kappa^4\eta_2\phi^8 + 2\Lambda\Big), \\
m_B^2 &= \frac{\kappa}{2}\Big(- m^2\phi + \lambda\phi^3 + \tfrac{3}{4}\kappa^2\eta_1\phi^5 + \tfrac{1}{2}\kappa^4\eta_2\phi^7\Big), \\
m_C^2 &= -m^2 + 3\lambda\phi^2 + \tfrac{15}{4}\kappa^2\eta_1\phi^4 + \tfrac{7}{2}\kappa^4\eta_2\phi^6, \\
m_D^2 &= -m^2 + \lambda\phi^2 + \tfrac{3}{4}\kappa^2\eta_1\phi^4 + \tfrac{1}{2}\kappa^4\eta_2\phi^6\,.
\end{aligned} \tag{6}
$$

We see that the graviton obtains a small mass m_A generated by the Higgs field. While beyond the scope of this letter, it would be important to further analyze the SM coupled to gravity in the light of different Higgs mechanisms for the graviton discussed in e.g. [16, 17].

Here we proceed by writing $\mathcal{L}^{\text{quad}}_{h_{\mu\nu},\text{H}} = -\frac{1}{2}V_I M^{IJ} V_J$ with the collective quantum field $V_I = (h_{\mu\nu}, \varphi_i, \psi_i)$. We perform the path integral at 1-loop order to find the gravitational contributions $V^{\text{(1-loop)}}_{\text{grav}} = \Delta V[\phi] - \Delta V[\phi]\Big|_{\kappa\to 0}$ to the effective 1-loop Higgs potential, where[3]

$$
\Delta V[\phi] = -\frac{i}{2}\bar{\mu}^{4-d}\int \frac{d^d p}{(2\pi)^d}\Big(9\ln(p^2 + m_A^2) + 3\ln(p^2 - m_D^2) + \ln[(p^2 + m_A^2)(p^2 - m_C^2) - 4m_B^4]\Big)\,. \tag{7}
$$

The relevant dimensionally regularized integral reads

$$
-\frac{i}{2}\,\bar{\mu}^{4-2\epsilon}\int \frac{d^{4-2\epsilon}p}{(2\pi)^{4-2\epsilon}}\ln(p^2 - m^2) = -\frac{m^4}{64\pi^2}\frac{1}{\epsilon} + \frac{m^4}{64\pi^2}\left[\ln\left(\frac{m^2}{\mu^2}\right) - \frac{3}{2}\right], \tag{8}
$$

[3]We have $\det M = 64(-\partial^2 + m_A^2)^9\,[(-\partial^2 + m_A^2)(\partial^2 + m_C^2) - 4m_B^4](\partial^2 + m_D^2)^3$.

with $\mu^2 = 4\pi\bar{\mu}^2 e^{-\gamma_E}$[4]. The pole in ϵ yields a renormalization of the scalar field couplings m, λ, η_1 and η_2. Proceeding in the $\overline{\text{MS}}$ scheme gives the following gravitational contribution to the renormalized effective potential of the standard model:

$$\Delta V[\phi] = \frac{9}{64\pi^2}\, m_A^4 \left(\ln\frac{m_A^2}{\mu^2} - \frac{3}{2}\right) + 3\frac{m_D^4}{64\pi^2}\left(\ln\frac{m_D^2}{\mu^2} - \frac{3}{2}\right) + \sum_{i=\pm}\frac{C_i^2}{64\pi^2}\left(\ln\frac{C_i}{\mu^2} - \frac{3}{2}\right), \tag{9}$$

where for conciseness we have defined[5]

$$C_\pm = \frac{1}{2}\left(m_C^2 - m_A^2 \pm \sqrt{\left(m_C^2 + m_A^2\right)^2 - 16 m_B^4}\right). \tag{10}$$

The $\boldsymbol{\beta}$-functions. Adding the counterterms necessary to absorb the $1/\epsilon$-pole terms to the bare Lagrangian yields the renormalized Lagrangian. An equivalent statement is that the effective potential obeys a renormalization group equation (RGE)

$$\left(\mu\tfrac{\partial}{\partial\mu} + \textstyle\sum_i \beta_i \tfrac{\partial}{\partial\lambda_i} - \gamma_\phi\, \phi\tfrac{\partial}{\partial\phi}\right) V_{\text{eff}}(\phi) = 0\,, \tag{11}$$

where $\beta_{\lambda_i} = \frac{d\lambda_i}{d\log\mu}$ are the β-functions of the couplings λ_i and γ_ϕ is the anomalous dimension of the SM Higgs field. From (9) one thus establishes the one-loop β-functions of the novel couplings $\eta_{1/2}$, which take the following form in the $\{\kappa^2 m^2, \kappa^4\Lambda\} \to 0$ limit:

$$\begin{aligned}
\beta^{(1)}_{\eta_1} &= 6\eta_1\gamma^{(1)}_\phi + \tfrac{1}{16\pi^2}\Big[108\,\lambda\,\eta_1 - 8\lambda^2\Big], \\
\beta^{(1)}_{\eta_2} &= 8\eta_2\gamma^{(1)}_\phi + \tfrac{1}{16\pi^2}\Big[192\lambda\,\eta_2 + 126\eta_1^2 + \tfrac{5}{4}\lambda^2 - 24\eta_1\lambda\Big], \\
\gamma^{(1)}_\phi &= \tfrac{1}{16\pi^2}\Big[3y_t^2 - \tfrac{3}{4}g_1^2 - \tfrac{9}{4}g_2^2\Big].
\end{aligned} \tag{12}$$

The scaling dimension γ_ϕ contains the top-Yukawa and electroweak coupling constants y_t, g_1, g_2. We stress that the λ^2 term in $\beta^{(1)}_{\eta_1}$ as well as the λ^2 and $\eta_1\lambda$ terms in $\beta^{(1)}_{\eta_2}$ are quantum gravity induced contributions despite the fact that they are not proportional to κ, see Figure 2. The remaining non-gravitational term in $\beta^{(1)}_{\eta_1}$ has been reported in [19], the non-gravitational terms in $\beta^{(1)}_{\eta_2}$ were considered in [20], however we disagree with the numerical factor for the $\lambda\eta_2$ term given there. We hence see that even in the absence of higher dimensional operators ($\eta_1 = \eta_2 = 0$) at an initial scale, these terms will be created in the renormalization group flow.[6] Including the Higgs

[4] We stress the use of dimensional regularization here, which only sees logarithmic divergences. A (naive) cutoff regularization $|p| < \Lambda_{\text{UV}}$ of the integral would also induce a $\Lambda_{\text{UV}}^2 m^2$ power divergence in (8). However, this term does not contribute to the β-functions in perturbatively renormalized effective field theory. Nevertheless, in the framework of the functional renormalization group approach this term contributes, c.f. [18].

[5] Note that the term under the square root is always positive at $\phi = \phi_{\min}$ on the parameter space of $\eta_{1,2}$.

[6] We assume that contributions to the RG-flow from other higher dimensional operators in the effective field theory are consistently neglectable at this order of perturbation theory. It would be important to investigate this point in full detail, cf. e.g. [19, 21].

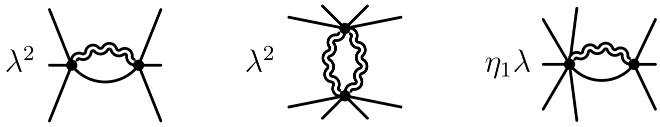

Figure 2: Diagrams of gravitational processes contributing to the β-functions of η_1 and η_2 in (12). Plain lines represent the Higgs field, wiggly lines the gravitons.

mass and cosmological constant in the analysis leads to quantum gravitational contributions of order $\kappa^2 m^2$ and $\kappa^4 \Lambda$ to all the β-functions including β_λ and β_m. However, these terms are of order 10^{-16} or less and absolutely negligible. We therefore set $m = \Lambda = 0$ in the remaining analysis. This also puts the electroweak minimum to zero, i.e. $V_{\rm ew} = 0$.

Consistent perturbation theory. The renormalization group improved SM effective potential is traditionally written in the form

$$V_{\rm SM}(\phi) = \lambda_{\rm eff}(\mu = \phi)\frac{\phi^4}{4}, \tag{13}$$

with the effective field-dependent coupling constant

$$\lambda_{\rm eff}(\phi) = e^{4\Gamma(\phi)}\Big[\lambda(\mu) + \lambda^{(1)}_{\rm eff}(\mu) + \lambda^{(2)}_{\rm eff}(\mu)\Big]\Big|_{\mu=\phi}, \tag{14}$$

where $\Gamma(\phi) = \int_{m_t}^{\phi} \gamma_\phi(\mu) d\log\mu$. The explicit expressions for the corrections to the effective coupling $\lambda_{\rm eff}$ up to three loops are given in the appendix of [4]. We follow a consistent use of perturbation theory along the lines of [22, 23, 11] assuming $\lambda \sim \hbar$. This is necessary in order for the tree-level $\lambda\phi^4$ term to receive non-negligible corrections by the one-loop contribution scaling as $y_t^4 \sim \hbar$. Perturbation theory in $\hbar$ is applicable, however, it is not the usual loop expansion. The expansion of the tree, one- and two-loop contributions of (13) along these lines yields the expansion up to next-to-leading order:

$$V^{\rm NLO}_{\rm SM}(\phi) = V^{\rm LO}_{\rm SM}(\phi) + V^{\rm (NLO)}_{\rm SM}(\phi)\,, \tag{15}$$

where $V^{\rm LO}_{\rm SM}$ scales as $\hbar$ and $V^{\rm (NLO)}_{\rm SM}$ as $\hbar^2$.

Let us now add the gravitational contributions to this picture. In analogy to the SM case above we write

$$V_{\rm grav}(\phi) = \eta_{\rm eff}(\mu = \phi)\frac{\phi^4}{4}, \tag{16}$$

with the RG-improved effective coupling

$$\eta_{\rm eff}(\phi) = \Big[e^{6\Gamma(\phi)}\eta_1(\mu)\kappa^2\phi^2 + e^{8\Gamma(\phi)}\eta_2(\mu)\kappa^4\phi^4 + \eta^{(1)}_{\rm eff}(\phi)\Big]\Big|_{\mu=\phi}. \tag{17}$$

Here $\eta^{(1)}_{\rm eff}(\phi) = 4V^{\text{(1-loop)}}_{\rm grav}/\phi^4$ can be extracted from (9). To obtain a consistent perturbative expansion we assume $\eta_1\kappa^2\phi^2 \sim \hbar$ and $\eta_2\kappa^4\phi^4 \sim \hbar$. Expanding $V_{\rm grav}(\phi)$ in $\hbar$ then yields the gravitational correction to the SM effective potential up to next-to-leading order $\hbar^2$:

$$V^{\rm NLO}_{\rm grav}(\phi) = V^{\rm LO}_{\rm grav}(\phi) + V^{\rm (NLO)}_{\rm grav}(\phi). \tag{18}$$

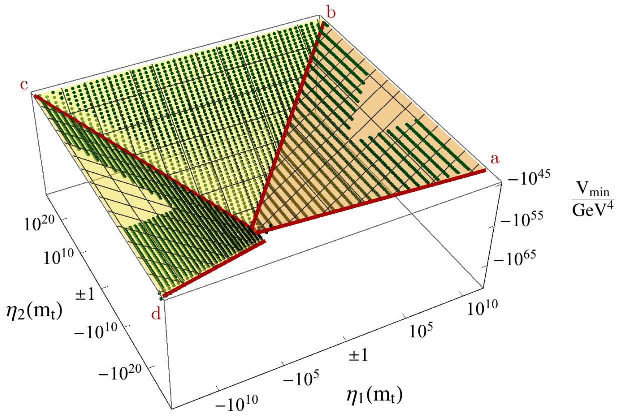

Figure 3: Plot of $V_{\min}$ for different initial values $\eta_1(m_\mathrm{t})$ and $\eta_2(m_\mathrm{t})$ at the top mass scale (green dots). In the triangular regions the huge ratio $\frac{\eta_1(m_\mathrm{t})}{\eta_2(m_\mathrm{t})}$ yields a breakdown of the numerical solution to the RGEs such that no data points are provided. The three (yellow) planes fit the data to good accuracy. At the boundaries of the $\eta_{1/2}$-plane, the value at the minimum becomes positive (before the minimum disappears).

Both, $V_{\mathrm{SM}}^{\mathrm{NLO}}$ and $V_{\mathrm{grav}}^{\mathrm{NLO}}$ have nonzero imaginary parts and in the following we will restrict to the real part of the potential, referring to [24] for an interpretation of the imaginary contribution.

Minimum of the effective potential. The full leading order (LO) potential reads[7]

$$V^{\mathrm{LO}}(\phi) = V^{\mathrm{tree}}(\phi) - \frac{3}{64} y_t^4\, \phi^4 \log \frac{y_t^2 \phi^2}{2\mu^2} + (g_1, g_2 \text{ terms}) \,,$$

where we suppress the numerically small gauge coupling terms for brevity. The position of the true minimum $\phi_{\min} = \mu_{\min}$ is then the second nontrivial solution of $\frac{d}{d\phi} V^{\mathrm{LO}} = 0$ with flowing couplings, guaranteeing gauge invariance of the minimum, c.f. [23, 11]. At NLO for the full effective potential we then have

$$V_{\min} = V^{\mathrm{LO}}(\phi_{\min}) + V_{\mathrm{SM}}^{(\mathrm{NLO})}(\phi_{\min}) + V_{\mathrm{grav}}^{(\mathrm{NLO})}(\phi_{\min}) \,. \tag{19}$$

In Figure 3 we plot the explicit data for $V_{\min}$ as a function of the initial values $\eta_1(m_\mathrm{t})$ and $\eta_2(m_\mathrm{t})$ at the top mass scale. Here we use the initial conditions provided in [4, 11], in particular $m_\mathrm{t} = 173.34$ GeV and $m_\mathrm{H} = 125.14$ GeV, the available higher loop RG equations for the SM and our one-loop RGEs for $\eta_{1/2}$. In large regions of the parameter space, the value at the minimum is well approximated by the three planes depicted in the double logarithmic plot in Figure 3 (bounded by the four (red) lines):

$$V_{\min}^{\mathrm{ab}} \simeq -\frac{2}{\eta_1^2 \kappa^4 10^8} \,, \qquad V_{\min}^{\mathrm{bc}} \simeq -\frac{1}{\eta_2 \kappa^4 10^5} \,, \qquad V_{\min}^{\mathrm{cd}} \simeq -\frac{1}{\eta_1^2 \kappa^4 10^5} \,.$$

[7] See [25] for a good review on effective potentials in the context of the SM.

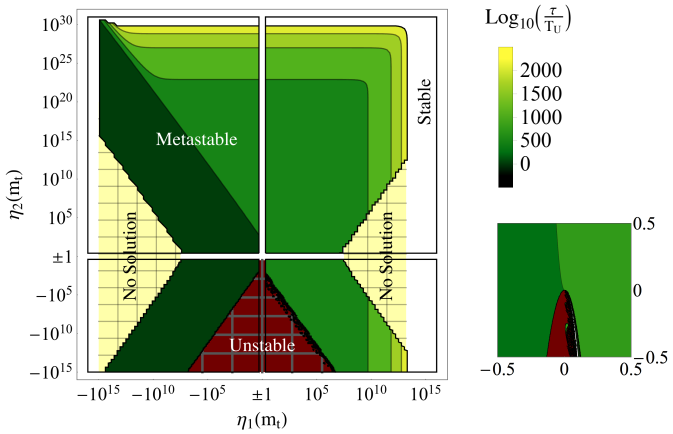

Figure 4: Portrait of the Higgs vacuum lifetime as a function of $\eta_{1/2}$ at the top mass scale. The dark meshed (red) region denotes instability; the light meshed (yellow) region could not be explored due to numerical issues. The white areas at the boundaries of the plot denote the stability region. On the right hand side we magnify the region $|\eta_{1/2}| \leq 0.5$.

Here the lines bounding the planes are parametrized by (a=d) $\eta_2 = -30\eta_1^2$, (b) $\eta_2 = 500\eta_1^2$ for $\eta_1 > 0$, (c) $\eta_2 = -\eta_1^2$ for $\eta_1 < 0$. Similar values were reported in [11] for $\eta_2 \equiv 0$ and non-flowing η_1. The minimum disappears approximately on the lines $\eta_1(m_t) \simeq 10^{14}$ and $\eta_2(m_t) \simeq 10^{31}$ in the $\eta_{1/2}$-plane. The value $V_{\min}$ turns positive shortly before, at around $\eta_1(m_t) \simeq 10^{12}$ and $\eta_2(m_t) \simeq 10^{30}$. Such astronomically high values for $\eta_{1/2}$ should be read as measures of scales M when new physics arises in the sense of $M = M_{\mathrm{Pl}}/\sqrt{\eta_1}$ and $M = M_{\mathrm{Pl}}/\eta_2^{1/4}$. In that sense the above thresholds reflect the existence of an intriguing stability scale $M \sim 10^{10}$ GeV. Finally, for positive $\eta_{1/2}(m_t) \lesssim 0.01$ the minimum lies beyond the Planck scale.

Let us briefly discuss the order of magnitude of the NLO gravity contributions to the effective potential. The ratio $V_{\mathrm{grav}}^{(\mathrm{NLO})}(\phi_{\min})/V_{\min}$ evaluated at the minimum ranges between zero and ten percent in large regions of the parameter space. At the boundaries, for large initial $\eta_1(m_t)$ or $\eta_2(m_t)$, the minimal value $V_{\min}$ changes sign as indicated above and the relative gravitational contribution becomes large.

Impact on vacuum stability. We have seen that for a huge range of the parameter space of $\eta_{1/2}$, the SM coupled to gravity develops a negative minimum $V_{\min}$ below the electroweak one. The lifetime τ in units of the age of the universe T_U of the false vacuum is usually estimated in the SM by [26] $\frac{\tau}{T_U} = \min_\mu \frac{1}{(\mu T_U)^4} \exp[\frac{8\pi^2}{3|\lambda(\mu)|}]$. In the presence of the dimension six and eight counterterms this equation needs to be modified. Numerical studies [27, 28] indicate that τ is still well approximated by simply replacing $|\lambda(\mu)|$ in the exponent of the lifetime expression by $|\lambda(\mu) + \eta_{\mathrm{eff}}(\mu)|$. In Figure 4 we show the lifetime of the false electroweak vacuum as a function of the initial values for $\eta_{1/2}$ at the top mass scale using this lifetime approximation. We see that

the SM coupled to gravity generically yields a highly metastable vacuum for a huge part of the parameter space in $\eta_{1/2}$. Notably, the instability only occurs for negative η_2, i.e. $\eta_2 < -30\eta_1^2$. Moreover, we find that the ultrashort lifetimes reported in [27] are confined to a very small (black) region in $\eta_{1/2}(m_t)$ parameter space once the RG-flow is taken into account.

Certainly the lifetime formula employed should be taken cautiously as it does not include curvature effects [15, 29]. In regions where $|V_{\min}|^{1/4}$ comes close to $M_{\rm Pl}$ the assumption of a flat background metric turns inconsistent. In these regions a full analysis expanding the metric around curved backgrounds should be employed (cf. e.g. [30] in this context).

Beyond the instability region derived above the values of the novel couplings η_1 and η_2 are not restricted by present observational data, as so far only single Higgs interactions have been probed experimentally (see e.g. [31] for a recent analysis)[8]. Therefore also the exponentially large values explored here are not excluded and remain perturbative.

Finally, we note that the reported results on the values of $V_{\min}$ and the lifetimes are sensitive to the values of $m_{\rm t}$ and $m_{\rm H}$ even at the level of a 2 GeV variation. Mapping this out is left for future work.

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